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Quantized Space-Time and Consequences

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We present a concrete quantized space-time at small distances. After transition to the large scale of nonquantized space-time, this method gives rise to a changed momentum operator which in turn leads to new infinite order differential equations describing extended (nonlocal) fields. The Green's functions of these equations are finite in the Euclidean momentum space, which provides for the construction of the theory of interacting fields free from ultraviolet divergences. In our scheme, interaction laws (e.g., the Coulomb, Yukawa potentials) between two particles are changed and have an attractive nature at small distances. As an example of this, finite quantum electrodynamics is constructed within the framework of quantized space-time. Restrictions on the parameter l of the theory are obtained: $l \lesssim 10^{-16}$ cm. Our scheme contains some interesting possibilities: description of quarks, gluons and tachyon-type objects, and indication of a way to a solution of the problem of quantization of particle mass and of quark confinement. Moreover, within the model one can obtain the scales $E_{\mathrm{EW}} \sim 118.1$ GeV and $E_{\mathrm{NW}} \sim 5353$ GeV of the unification of electromagnetic and weak, and weak and nuclear processes, respectively. The last possibility is very interesting for the experimental verification of the theory.

1. INTRODUCTION

The problem of space-time structure is always an important part of any physical theory. Starting with Newtonian times the concept of space-time and the interrelation between its structure and properties of matter has received much attention and developments, and arrived at its peak due to Einstein's genius. It transpires that the pure mathematical and geometrical concept of space and time became the natural arena of physical phenomena. For example, Einstein and Minkowski united space and time as a single entity and introduced the new concept of the space-time manifold—the pseudo-Euclidean four-dimensional space which was a starting point for

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the general theory of relativity and modern relativistic quantum field theory. Further, states of quantum mechanical systems are described by introducing an infinite-dimensional functional space, the Hilbert space, while for the description of the sub-nucleon processes it was necessary to introduce, simultaneously with the usual space-time, supplementary so-called internal abstract spaces connected with quantum numbers (spin, isospin, strangeness, charge, etc.) of the elementary particles. This abstract concept of spaces turned out to be very useful and enduring in classifications of the elementary particles and in the unified description of all types of their interactions. In the latter case, a beautiful example is the electroweak theory due to Weinberg (1967), Salam (1968), and Glashow (1961).

Recently, the concepts of superspace and supersymmetry (see, for example, reviews of Van Nieuwenhuizen, 1981; Zumino, 1983 and also Vladimirov and Volovich, 1984) and pregeometry (Wheeler, 1964; Akama, 1981; Terazawa, 1981), the submicroscopic scale and lattice (discrete) structure of space-time (Wilson, 1974; Kogut and Susskind, 1975; Vialtsev, 1965) and stochastic (or fluctuational) (Prugovečki, 1984; Namsrai, 1985, and references therein) and higher-dimensional geometry (Snyder, 1947; Kadyshevsky, 1980) have been intensively studied as a base of future physical theories. Friedberg and Lee (1983) and Cole (1972), for example, discussed discrete quantum mechanics and a cellular space-time structure, respectively. Finkelstein (1969, 1972, 1974) published a series of papers developing a theory including space-time structure—the so-called space-time code. Prugovečki's (1984) monograph is devoted to a consistent unification of relativity and quantum theory based on stochastic spaces. In previous work (Namsrai, 1985) we have discussed an idea of the stochastic character of space-time at small distances, which has played the most important role in constructing the nonlocal theory (Efimov, 1977) of quantized fields and is given the very nature of stochastic quantum mechanics (Nelson, 1967; Guerra, 1981).

The theoretical and experimental successes of high-energy physics now dictate a deeper level of understanding of the structure of space-time and its properties at small distances. However, our concepts of space-time are confirmed experimentally to be valid to distances of the order 10^{-15}-10^{-16} cm (see below).

It is a fact that phenomena in the microworld are quantized, i.e., their properties are described by quantum laws, but connected with them space-time structure becomes continuous at least up to the above-mentioned distances. The structure of space-time and the physical phenomena within it enter inseparably into human cognition, and their interrelations are those of dialectical unity. This unity gives rise to some hope that the quantum nature of space-time properties can exist in the microworld and be discovered sooner or later. In this assumption the following question arises: At what distances does the quantum structure of space-time start? The majority of physicists believe that quantum gravitational effects occur at distances of the order of 10^{-33} cm. From the practical point of view this length is so small that the contribution made by the quantum gravitational effect to any physical quantities is in fact zero, i.e., there is no hope that this effect may be discovered by physical processes taking place in the microworld. However, it is not to be ruled out that between distances 10^{-33} cm and 10^{-16} cm there may exist some oasis in which the quantum structure of space-time may be manifested.

The historical idea of the quantum structure of space-time has been discussed by many authors, in particular, beginning from the early stages of the development of field theory and devoted to the construction of a finite theory of quantized fields free from ultraviolet divergences. In the theory of quantized space-time it is usually assumed that there is no exact conceptual meaning of definite space-time points, i.e., the components of the operator of coordinates are not commuted

$$[\hat{x}_\mu, \hat{x}_\nu] \neq 0 \qquad \text{for } \mu \neq \nu$$

The theory of quantized space-time was first discussed by Snyder (1947) and subsequently developed by Kadyshevsky (1959, 1962), Gol'fand (1959, 1962), and Tamm (1965). In their approaches, the symmetry between the geometrical structure of the x space and p space is violated but in the theory of quantized space-time proposed by Yang (1947), Born's reciprocity principle (symmetry between coordinate and momentum variables) is valid (see, also Leznov, 1967; Kirzhnits and Chechin, 1967). For discussion of various theoretical ideas of space-time structures in the microworld, where earlier references can be found, see Blokhintsev (1973) and Prugovečki (1984).

In this paper we present a concrete method of introducing quantized space-time into physical theory. It turns out that the proposed method gives the nonlocal stochastic theory of quantized fields considered in Efimov (1977) and Namsrai (1985). In our approach it is suggested that the coordinates $\hat{x}_\mu$ of events consist of two parts: external (usual) coordinates x_μ and internal coordinates r_μ; $\hat{x}_\mu = x_\mu + r_\mu$. As seen below, owing to the last terms our space-time is quantized at small distances and in accordance with this momentum operator is also changed. After transformation to the large scale of space-time, this change defines contributions to any observable quantities due to quantized space-time effects and gives a finite theory (free from ultraviolet divergences). Notice that this shift of coordinates recalls the description method of the bilocal system proposed by Katayama and Yukawa (1968) and Sogami (1973).

In Section 2 we introduce quantized space-time and realize its position in the case of massless particles. Section 3 is devoted to the generalization of the obtained method to massive particles. Section 4 deals with the construction of finite quantum electrodynamics based on the hypothesis of quantized space-time. In the last section we present some speculation and discussion of obtained results.

2. QUANTIZED SPACE-TIME AND MASSLESS PARTICLES

We suggest that our four-dimensional pseudo-Euclidean space-time R_4 becomes quantized at small distances and consists of two parts: external E_4 and internal I_4, $R_4 \Rightarrow \hat{R}_4 = E_4 + I_4$, i.e., the coordinates of events $\hat{x}_\mu \in \hat{R}_4$ have the form $\hat{x}_\mu = x_\mu + r_\mu$, where $x_\mu \in R_4$ ($R_4 = E_4$) and $r_\mu \in I_4$. Since, in our model the actual points of the space-time are of a quantized nature, these points cannot be used as a basis for a coordinate system, nor can one take a derivative with respect to them. However, the space-time of common experience (i.e., the laboratory frame) is nonquantized on a large scale. It is only in the microworld where the quantization manifests itself. One can then proceed mathematically from the microworld to this large-scale non-quantum space-time. This mathematical construction provides a nonquantum space-time to which the quantum physical space-time can be referred. We hypothesize that there exists some procedure of averaging on the internal space I_4, which guarantees such passage.

To realize the above, we now consider a concrete form of quantized space-time $\hat{R}_4$ with coordinates $\hat{x}_\mu = (\hat{x}_0, \hat{\mathbf{x}}) = (\hat{x}_0, \hat{x}, \hat{y}, \hat{z})$;

$$x_\mu \Rightarrow \hat{x}_\mu = x_\mu + l\gamma_\mu \tag{1}$$

where parameter l means a value of the fundamental length and γ_μ are Dirac γ matrices. Because of relations of γ matrices

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}, \qquad g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$$

we have the following rules for the coordinates $\hat{x}_\mu$:

$$[\hat{x}_\mu, \hat{x}_\nu] = \hat{x}_\mu\hat{x}_\nu - \hat{x}_\nu\hat{x}_\mu = 2il^2\sigma_{\mu\nu} \tag{2}$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}\frac{1}{i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

From these commutation relations we see immediately that $[\hat{x}_\mu, \hat{x}_\nu] \neq 0$, for $\mu \neq \nu$. This means that our space-time $\hat{R}_4$ is quantized at small distances.

Another realization may be chosen, that is,

$$\hat{x}_\mu = x_\mu + il\gamma_\mu \tag{3}$$

In this case, we have

$$[\hat{x}_\mu, \hat{x}_\nu] = \frac{2}{i} l^2 \sigma_{\mu\nu} \tag{4}$$

Further we assume that in accordance with shifts (1) and (3) the momentum operators $\hat{p}_\mu \Rightarrow \hat{\mathscr{P}}_\mu$ are changed at the same time. In particular, in the given model eigenfunctions and eigenvalues of momentum operators $\hat{\mathscr{P}}_\mu$ are given by equations

$$\hat{\mathscr{P}}_\mu \phi = \mathscr{P}_\mu \phi \tag{5}$$

where ϕ acquires the form

$$\phi = c \exp\{i p_\mu \hat{x}_\mu\} \tag{6}$$

in the $\hat{x}$ representation. Our problem is now to find the form of operators $\hat{\mathscr{P}}_\mu$ and their eigenvalues $\mathscr{P}_\mu$ and eigenfunctions ϕ at the large scale.

In accordance with the above deduction we must here construct a mathematical procedure which provides a passage to the large scale from the small one. In our case, it is natural that this procedure leads to the averaging of coordinates $r_\mu = l\gamma_\mu$ of the internal space I_4. Thus, as a result of such averaging, by tracing the γ matrices, we have

$$\langle \hat{x}_\mu \rangle_{\hat{R}_4} = x_\mu, \qquad \langle \hat{x}_\mu \hat{x}_\nu \rangle = x_\mu x_\nu + 4l^2 g_{\mu\nu}$$

In particular

$$\langle \hat{s}^2 \rangle = \langle \hat{x}_0^2 \rangle - \langle \hat{\mathbf{x}}^2 \rangle = s_0^2 + 16 l^2, \qquad s_0^2 = x_0^2 - \mathbf{x}^2 \tag{7}$$

Similarly, for the second case (3) we get

$$\langle \hat{x}_\mu \hat{x}_\nu \rangle = x_\mu x_\nu - 4l^2 g_{\mu\nu}, \qquad \langle \hat{s}^2 \rangle = s_0^2 - 16 l^2 \tag{8}$$

The explicit form of the eigenfunction (6) in the case of (1) is determined by the following expression:

$$\phi = c\, e^{ipx} f(\gamma_\mu p'_\mu) = c\, e^{ipx} \left\{ \frac{1}{2} [f(p') + f(-p')] + \frac{1}{2} \frac{p'_\mu \gamma_\mu}{p'} [f(p') - f(-p')] \right\} \tag{9}$$

where $p'_\mu = ilp_\mu$, $p' = (p'_\mu p'_\mu)^{1/2} = l(\mathbf{p}^2 - p_0^2)^{1/2}$, $f(p') = \exp(p')$. The formula (9) is easily verified by using the so-called Sylvester expansion for the matrix function (Frazer et al., 1952) as well as the obvious power series expansion

$$f(x+l\gamma)=f(x)+l\gamma_\mu\partial_\mu f(x)+\frac{l^2}{2!}\langle\gamma_\mu\gamma_\nu\rangle\partial_\mu\partial_\nu f(x)$$
$$+\frac{l^3}{3!}\langle\gamma_\mu\gamma_\nu\gamma_\rho\rangle\partial_\mu\partial_\nu\partial_\rho f(x)+\cdots$$

where $\langle\gamma_{\mu_1}\gamma_{\mu_2}\cdots\gamma_{\mu_n}\rangle=(1/n!)\sum\gamma_{\mu_1}\gamma_{\mu_2}\cdots\gamma_{\mu_n}$, the sum being taken over all the $n!$ permutations of the indices (see also Nellman, 1964). By virtue of our procedure of averaging, the eigenfunction (9) takes the form at the large-scale

$$\phi_R=\langle\phi\rangle_{\hat{R}_4}=c\cosh[l(\mathbf{p}^2-p_0^2)^{1/2}]\,e^{ipx}=c\cosh[l(-p^2)^{1/2}]\,e^{ipx} \tag{10}$$

or in the case of (3)

$$\phi_R=\langle\phi\rangle_{\hat{R}_4}=c\cosh[l(p^2)^{1/2}]\,e^{ipx},\qquad px=p_0x_0-\mathbf{px} \tag{11}$$

If taking into account the fact that $\cosh x$ is the entire analytic function decomposed by power series over the even orders of $(x^{2k}/2k!)$ one can rewrite expressions (10) and (11) in the following form:

$$\phi_R=c\cosh[l(\mp\Box)^{1/2}]\,e^{ipx},\qquad \Box=-\partial^2/\partial x_0^2+\partial^2/\partial\mathbf{x}^2$$

where the upper and lower sign corresponds to the case (1) and (3), respectively. Now it is not difficult to obtain the explicit form of the operator $\hat{\mathscr{P}}_\mu$ and its eigenvalue $\mathscr{P}_\mu$; they are:

$$\hat{\mathscr{P}}_\mu=\hat{p}_\mu\cosh[l(\mp\Box)^{1/2}],\qquad \mathscr{P}_\mu=p_\mu\cosh[l(\mp p^2)^{1/2}]$$

here $\hat{p}_\mu=-i\hbar\partial/\partial x_\mu$.

By the usual rule, in our model a free Lagrangian of the field $\varphi_R(x)$ is constructed in terms of the quadratic form $\varphi_R\hat{\mathscr{P}}^2\varphi_R$ of the operator $\hat{\mathscr{P}}^2$.

For example, in the case of the massless scalar field, we have

$$\mathscr{L}=\tfrac{1}{2}\varphi_R(x)\hat{\mathscr{P}}^2\varphi_R(x) \tag{12}$$

The equation of motion of this field is obtained in the usual manner (the principle of stationary action)

$$\Box\cdot\cosh^2[l(\mp\Box)^{1/2}]\varphi_R(x)=0 \tag{13}$$

For the photon field $A_\mu^R(x)$ without the source field $J_\mu(x)$:

$$\begin{gathered}\mathscr{L}_{\mathrm{em}}=-\tfrac{1}{2}[\hat{\mathscr{P}}_\nu A_\mu^R(x)]^2\\ \Box\cdot\cosh^2[l(\mp\Box)^{1/2}]A_\mu^R(x)=0\end{gathered} \tag{14}$$

The application of such a choice of Lagrangian form for the electromagnetic field has been discussed by 't Hooft and Veltman (1973).

We see that these equations are differential equations of infinite order, i.e., they are in fact integral equations. In order to solve the Cauchy problem we have to know the values of the function $\varphi_R(x)$ and all its derivatives in the initial moment of time. Thus, unlike usual fields obeying differential equations of finite order (in most cases, it is the second order), we obtain new objects, nonlocal (extended) fields of the Efimov (1977) type. We denote these objects by subindex R; for example, $\varphi_R(x)$ and $A_\mu^R(x)$ are extended scalar and photonlike fields, respectively.

Now we study equation (14) in the momentum representation:

$$p^2 \cosh^2[l(\mp p^2)^{1/2}]\tilde{A}_\mu^R(p)=0 \tag{15}$$

where $\tilde{A}_\mu^R(p)$ is the Fourier transform of $A_\mu^R(x)$,

$$\tilde{A}_\mu^R(p)=(2\pi)^{-4}\int d^4x\, e^{-ipx}A_\mu^R(x) \tag{16}$$

Equation (15) has two solutions:

$$\begin{aligned} &(1)\quad p^2=0 \qquad \text{since } \cosh 0=0\\ &(2)\quad p^2\neq 0 \qquad \text{but } \cosh^2[l(\mp p^2)^{1/2}]=0 \end{aligned} \tag{17}$$

In the latter case two cases (1) and (3) should be distinguished. In the former one we have for $p^2>0$

$$\cos[l(p^2)^{1/2}]=0$$

or

$$(p^2)^{1/2}=M_n=\frac{1}{l}\pi(\tfrac{1}{2}+n), \qquad n=0,1,2,3,\ldots \tag{18}$$

It is easily seen that the case (3) does not work for the physical state $p^2>0$, but tachyons type of solutions may exist, i.e., hypothetical particles with complex masses, or alternatively strange cases of complex length $l\to il$. In any case, both these solutions are ruled out of our consideration.

Thus, we observe the interesting fact that a hypothesis of quantized space-time leads to the generation of a family of particles or to a mechanism resembling particles with mass. It means that a massless particle (e.g., photon) helps to generate other massive particles at the price of the loss of its freedom ($p^2\neq 0$) in quantized space-time at small distances. Notice that in the case of (3) we have only one solution $p^2=0$ in (17), therefore, in this case a family of particles does not appear.

Since $\cosh[l(-p^2)^{1/2}]=1$, for $p^2=0$, we see that the eigenvalues $\mathscr{P}_\mu = p_\mu \cosh[l(-p^2)^{1/2}]$ of the momentum operator $\hat{\mathscr{P}}_\mu$ coincide with p_μ in the free particle case, i.e., the following formal equality

$$p_\mu \cosh[l(-p^2)^{1/2}]\tilde{A}^R_{\mu,F}(p) = p_\mu \tilde{A}_{\mu,F}(p) \tag{19}$$

holds in the plane wave case, where $\tilde{A}_{\mu,F}$ is the Fourier transform of the free local field $A_{\mu,F}(x)$ satisfying equation $\Box A_{\mu,F}(x) = 0$. Further, we assume that an equation of the type of (19) is valid in the case $p^2 \neq 0$, i.e., formal relations

$$\tilde{A}^R_\mu(p) = \cosh^{-1}[l(-p^2)^{1/2}]\tilde{A}_\mu(p) \quad \text{and} \quad A^R_\mu(x) = \cosh^{-1}[l(-\Box)^{1/2}]A_\mu(x) \tag{20}$$

can be applied analytically to any variables $p_\mu = p_0, \mathbf{p}$ ($p^2 \neq 0$) and $x_\mu = x_0, \mathbf{x}$, respectively.

Thus, our formalism coincides with the usual scheme of quantum field theory in the free particle case and therefore gives no new information in the given case. However, in the virtual states of particles both the formalisms are essentially different. We now turn to this situation. The main object of the virtual state of the particle is its Green's function (or propagator). As is well known (see, for example, 't Hooft and Veltmann, 1973), the propagators are minus the inverse of the operators found in the quadratic term of the free Lagrangian, for example,

$$\mathscr{L} = \tfrac{1}{2}\varphi(x)[\Box - m^2]\varphi(x) \Rightarrow (m^2 - k^2 - i\varepsilon)^{-1}$$

This rule reads for the Lagrangian (14):

$$\tilde{D}^R_{\mu\nu}(k) = g_{\mu\nu}/\{(-k^2 - i\varepsilon)\cdot \cosh^2[l(-k^2)^{1/2}]\} \tag{21}$$

or very simply the propagator (21) is defined in terms of the equality (20) [bearing in mind that the T-ordering symbol concerns the field $A^R_\mu(x)$]

$$\begin{aligned} D^R_{\mu\nu}(x-y) &= \langle 0|\, T\{A^R_\mu(x)A^R_\nu(y)\}|0\rangle \\ &= \cosh^{-2}[l(-\Box_x)^{1/2}]\langle 0|\, T\{A_\mu(x)A_\nu(y)\}|0\rangle \\ &= \cosh^{-2}[l(-\Box_x)^{1/2}]D_{\mu\nu}(x-y) \end{aligned}$$

where $D_{\mu\nu}(x)$ is the usual local Green's function of the photon field. In the momentum representation the latter expression gives

$$\tilde{D}^R_{\mu\nu}(k) = \frac{g_{\mu\nu}}{-k^2 - i\varepsilon}\cosh^{-2}[l(-k^2)^{1/2}] \tag{22}$$

On the other hand, the Green's function $D^R_{\mu\nu}(x)$ is the solution of the following equation:

$$\Box \cdot \cosh^2[l(-\Box)^{1/2}]D^R_{\mu\nu}(x) = -g_{\mu\nu}\delta^4(x) \tag{23}$$

The solution [the causal Green's function $D^R_{\mu\nu}(x)$] to this equation is given by the counter integral:

$$D^R_{\mu\nu}(x) = -g_{\mu\nu} D^R(x) = -\frac{g_{\mu\nu}}{(2\pi)^4 i} \int_c d^4k \frac{e^{-ikx}}{-k^2 - i\varepsilon} \cosh^{-2}[l(-k^2)^{1/2}] \tag{24}$$

The counter integration c is chosen as the usual local theory and is determined by the "$i\varepsilon$ rule" (see Figure 2).

It is important to notice that in our scheme ultraviolet divergences are absent, since $D^R_{\mu\nu}(0) < 0$, for example,

$$D^R(0) = \frac{-1}{(2\pi)^4 i} \int_c \frac{d^4k}{-k^2 - i\varepsilon} \cosh^{-2}[l(-k^2)^{1/2}] < 0$$

Indeed, after transformation to the Euclidean metric we get

$$D^R(0) = \frac{-\pi^2}{(2\pi)^4} \int_0^\infty du \cosh^{-2}(l\sqrt{u}) = -\frac{2\pi^2}{(2\pi)^4} \int_0^\infty dx\, x \cosh^{-2}(lx)$$

$$= -\frac{1}{8} \frac{\ln 2}{\pi^2 l^2}$$

At the same time as the photon propagator (24), the Coulomb law is also changed. Thus, the potential of the two interacting charged particles acquires the form in the static limit:

$$U_C(r) = \frac{e^2}{(2\pi)^3} \int d^3p \frac{e^{i\mathbf{pr}}}{\mathbf{p}^2} \cosh^{-2}[l(\mathbf{p}^2)^{1/2}] = \frac{e^2}{2\pi^2 r} \int_0^\infty \frac{dx}{x} \sin(xr) \cosh^{-2}(lx) \tag{25}$$

From this, it is easy to see that $U_C(0) < 0$, indeed

$$U_C(0) = (e^2/2\pi^2) \int_0^\infty dx \cosh^{-2}(lx) = [-1/2\pi^2 l] e^2$$

We now give the Mellin representation for the propagator $\tilde{D}^R_{\mu\nu}(k)$ of the photon field. For this, making use of the expansion for $\cosh^{-2} x$,

$$\cosh^{-2} x = 4/(e^{2x} + 2 + e^{-2x}) = -4 \sum_{n=1}^{\infty} (-1)^n n\, e^{-2nx} \tag{26}$$

we get

$$\cosh^{-2} x = 4 \sum_{n=1}^{\infty} (-1)^{n+1} n \sum_{k=0}^{\infty} (-1)^k \frac{(2nx)^k}{k!} = 4 \sum_{n=1}^{\infty} (-1)^{n+1} n$$

$$\times \frac{1}{2i} \int_{-\beta+i\infty}^{-\beta-i\infty} \frac{d\rho (2nx)^\rho}{\sin \pi\rho \cdot \Gamma(1+\rho)} \tag{27}$$

where $1 > \text{Re}\,\rho > 0$. Using the properties of the $\Gamma(x)$ function it is possible to move the counter integration in the expression (27) to the left through the point $\rho = -1$ and in obtained results one can take

$$\sum_{n=1}^{\infty} (-1)^{n+1} n^{1+\rho} = (1 - 2^{2+\rho})\zeta(-1-\rho)$$

since $\text{Re}(-1-\rho) > 0$, where $\zeta(z)$ is the Riemann ζ function having the single pole at the point $z = 1$ and satisfying the following conditions:

$$2^{1-z}\Gamma(z)\zeta(z)\cos\tfrac{1}{2}\pi z = \pi^z \zeta(1-z)$$

$$\zeta(2m) = 2^{2m-1}\pi^{2m}\frac{|B_{2m}|}{(2m)!}$$

$$\zeta(-2m) = 0$$

$$\zeta(1-2m) = -B_{2m}/2m, \qquad m = 1, 2, 3, \ldots$$

in particular,

$$\zeta(0) = \tfrac{1}{2}, \qquad \zeta(-1) = -B_2/2 = -\tfrac{1}{12}$$

Here B_m are the Bernoulli numbers, for example

$$B_0 = 1, \qquad B_1 = -\tfrac{1}{2}, \qquad B_2 = \tfrac{1}{6}$$

As a result, we obtain

$$\cosh^{-2} x = \frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho \frac{v(\rho)}{\sin \pi\rho} \frac{x^\rho}{\Gamma(1+\rho)} \qquad (2 < \gamma < 1) \tag{28}$$

where

$$v(\rho) = 4 \cdot 2^\rho (1 - 2^{2+\rho})\zeta(-1-\rho) \tag{29}$$

In particular,

$$v(-1) = -1, \qquad v(0) = 1, \qquad v(1) = 0, \qquad \text{and } v(2) = 2$$

After these simple calculations we have the following Mellin representation for the photon propagator:

$$\tilde{D}^R_{\mu\nu}(k) = -\frac{g_{\mu\nu}}{2i}\int_{\substack{-\gamma+i\infty \\ (2<\gamma<1)}}^{-\gamma-i\infty} d\rho \frac{v(\rho)}{\sin \pi\rho\Gamma(1+\rho)} l^\rho[-k^2 - i\varepsilon]^{(1/2)\rho - 1} \tag{30}$$

Representations (28) and (30) are very convenient for the purpose of concrete calculation. For example, by using the representation (28) the expression (25) for the potential $U_C(r)$ is calculated explicitly and takes the form

$$U_C(r)=\frac{1}{2\pi^2 r}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\frac{v(\rho)}{\sin\pi\rho\Gamma(1+\rho)}\sin\tfrac{1}{2}\pi\rho\cdot\Gamma(\rho)\left(\frac{l}{r}\right)^{\rho}$$

$$=\frac{1}{2\pi^2 r}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty}\frac{d\rho\, v(\rho)}{2\rho\cos\frac{1}{2}\pi\rho}\left(\frac{l}{r}\right)^{\rho} \tag{31}$$

Here the following integral is used:

$$\int_0^\infty dx\, x^{\rho-1}\sin ax = a^{-\rho}\cdot\Gamma(\rho)\sin\tfrac{1}{2}\pi\rho, \qquad [a>0, 0<|\mathrm{Re}\,\rho|<1]$$

Two cases should be distinguished: $(l/r)>1$ (i.e., $r\to 0$) and $(l/r)<1$ $(r\to\infty)$. In both first and second cases it is necessary to move the counter integration in (31) to the left and to the right, respectively. Thus, we have

$$U_C(r)=-\frac{e^2}{2\pi^2 r}[-v(-1)(l/r)^{-1}+\tfrac{1}{3}v(-3)(l/r)^{-3}+O(r/l)^5]$$

$$=-\frac{e^2}{2\pi^2 l}-\frac{1}{16\cdot 9}e^2\frac{r^2}{l^3}+O(r^4/l^5) \qquad \text{for } r<l$$

and

$$U_C(r)=\frac{e^2}{4\pi r} \qquad \text{for } l<r \tag{32}$$

The function $U_C(r)$ at $r=0$ represents the so-called proper electrostatical energy of the electron in the classical field theory. As seen above, in our model the proper energy of the electron is finite $U_C(0)\sim e^2/l$. This result coincides with the well-known classical electrodynamical value $U_C(0)\sim e^2/a$, where a is the electron size. In the last case it is usually assumed that the electron is a pointlike object with the radius a. However, in our case there is an interesting possibility: Because of the minus sign of $U_C(r)$ for $r=0$ (see Figure 1) two electrons may form a whole bound state, i.e., unlike the usual classical theory, in quantized space-time the electric repulsion between two electrons turns to electric attraction at small distances.

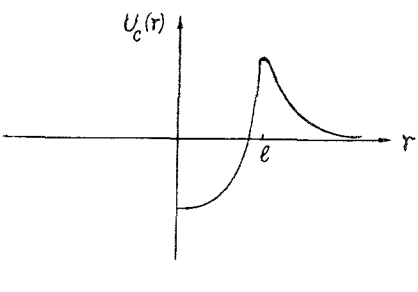

Fig. 1. The illustration of the change of the Coulomb potential due to quantized space-time at small distances.

On the other hand, at distances $r > l$ our potential $U_C(r)$ reproduces exactly the Coulomb law (without any terms of the type $e^2(l^2/r^3), \ldots$). This means that quantum electrodynamics is a beautiful local theory up to distances l, if $l \sim 10^{-33}$ cm then QED becomes local once and for all.

3. BEHAVIOR OF MASSIVE PARTICLES IN QUANTIZED SPACE-TIME

From the above consideration we see that the particular form (1) of introducing quantized space-time gives rise to the generation of families of particles with mass and we therefore believe that in general cases the hypothesis of quantized space-time may be used to describe a system with variable masses. So, it leads to the introduction of five variables $x_0, \mathbf{x}, s$ conjugated to p_0, $\mathbf{p}$, m in the intermediate stage of calculations, where s and m are not constants and change in a continuous manner. Roughly speaking, s may be understood as a proper time variable of particles. In this case the fundamental quadratic forms $\hat{s}^2$ and $\hat{p}^2$ take the form

$$\hat{s}^2 = \delta_{ij}x_ix_j = x_0^2 - \mathbf{x}^2 - s^2 \qquad \text{and} \qquad \hat{p}^2 = \delta_{ij}p_ip_j = p_0^2 - \mathbf{p}^2 - m^2,$$

$$i, j = 0, 1, 2, 3, 4, \qquad \delta_{00} = -\delta_{11} = -\delta_{22} = -\delta_{33} = -\delta_{44} = 1$$

In connection with dimensional regularization, some generalization of combinatoric rules was carried out. In particular, the following rules are valid in $n = 2\omega$-dimensional space (see, for example, Leibbrandt, 1975)

$$\begin{aligned} &\delta_{ij}p_j = p_i, \qquad p_ip_i = p^2, \qquad \delta_{ij}\delta_{ik} = \delta_{jk} \\ &\delta_{ij}\delta_{ij} = 2\omega, \qquad \delta_{ii} = 2\omega \\ &\{\gamma_i, \gamma_j\} = 2\delta_{ij}I, \qquad I = \text{unit matrix} \\ &\mathrm{Tr}(\gamma_i\gamma_j) = 2^{\omega}\delta_{ij} \end{aligned} \tag{33}$$

It should be noted that for our purpose it is sufficient to use only the commutation rule (33). Since, after averaging the procedure on quantized space-time we must turn to the usual case $m \to m_0$ and $s \to s_0$ (s_0, m_0 are constants) by taking $\psi(x, s) = \delta(s - s_0)\psi(x)$ and $\tilde{\psi}(p, m) = \delta(m - m_0)\tilde{\psi}(p)$.

Owing to relation (33) all above-obtained formulas (2), (4)-(6), and (9) preserve their forms, there $px = p_0x_0 - \mathbf{px} - sm$. But in the case of (1) the expression (10) and values of $\hat{\mathscr{P}}_\mu$, $\mathscr{P}_\mu$, and $\hat{\mathscr{P}}^2$ take the forms after transition to four-dimensional space-time

$$\begin{aligned} \phi_R &= \{c \cosh[l(m^2 - p^2)^{1/2}] \\ &= c \cosh[l(m^2 - p_0^2 + \mathbf{p}^2)^{1/2}]\}\, e^{ipx} \qquad (px = p_0x_0 - \mathbf{px}) \end{aligned}$$

$$\hat{\mathscr{P}}_i = \hat{p}_i \cosh[l(m^2 - \Box)^{1/2}], \qquad \hat{\mathscr{P}}_i = \left\{\hat{\mathscr{P}}_\mu, -i\frac{\partial}{\partial s}\right\}, \qquad \hat{p}_i = \left\{-i\frac{\partial}{\partial x_\mu}, m\right\}$$

$$\mathscr{P}_i = p_i \cosh[l(m^2 - p^2)^{1/2}], \; p_i = \{p_\mu, m\}, \; p_i p_i = p^2 - m^2 \; (\mu = 0, 1, 2, 3) \tag{34}$$

$$\hat{\mathscr{P}}^2_m = (\Box - m^2) \cosh^2[l(m^2 - \Box)^{1/2}], \qquad \hat{p}_i \hat{p}_i = \Box - m^2$$

Here the subindex zero for m_0 is omitted and the inessential multiplier $\exp(-im_0 s_0)$ has been absorbed in a constant c. This factor $\exp(-im_0 s_0)$ has arisen from the procedure of lowering the dimension of space-time and is given by

$$\exp(ipx - im_0 s_0) = \int\int ds\, dm\, \delta(s - s_0)\, \delta(m - m_0) \exp(ipx - ism)$$

Thus, at this stage a formal consideration of five-dimensional space-time is finished, but in our further study quantities (34) have an important role.

In the massive scalar particle case the free Lagrangian (12) is constructed by the operator $\hat{\mathscr{P}}^2_m$ and now acquires the form

$$\mathscr{L} = \tfrac{1}{2}\varphi_R(x)\hat{\mathscr{P}}^2_m \varphi_R(x) \tag{35}$$

Equation of motion (13) is given by

$$(\Box - m^2) \cosh^2[l(m^2 - \Box)^{1/2}]\varphi_R(x) = 0 \tag{36}$$

or

$$(p^2 - m^2) \cosh^2[l(m^2 - p^2)^{1/2}]\tilde{\varphi}_R(p) = 0 \tag{37}$$

In our model natural generalization may be made clearly in the fermion field case, for example, the Dirac equation has the following form in the momentum representation:

$$(\hat{p} - m) \cosh[l(m^2 - p^2)^{1/2}]\tilde{\psi}_R(p) = 0 \tag{38}$$

Green's functions $\tilde{D}^R_c(p)$ and $\tilde{S}^R_c(\hat{p})$ corresponding to the equations (37) and (38) are given by

$$\tilde{D}^R_c(p) = (m^2 - p^2 - i\varepsilon)^{-1} \cosh^{-2}[l(m^2 - p^2)^{1/2}] \tag{39}$$

and

$$\tilde{S}^R_c(\hat{p}) = \frac{m + \hat{p}}{m^2 - p^2 - i\varepsilon} \cosh^{-1}[l(m^2 - p^2)^{1/2}] \tag{40}$$

As in the massless case any physical theory with the propagators (39) and (40) is finite, i.e., free from ultraviolet divergences. It is clear that causal

Green's functions

$$D_c^R(x-y)=\langle 0|T[\varphi_R(x)\varphi_R(y)]|0\rangle=(2\pi)^{-4}i^{-1}\int d^4p\, e^{-ip(x-y)}\tilde{D}_c^R(p) \tag{41}$$

$$S_c^R(x-y)=\langle 0|T[\psi_R(x)\bar{\psi}_R(y)]|0\rangle=(2\pi)^{-4}i^{-1}\int d^4p\, e^{-ip(x-y)}\tilde{S}_c^R(\hat{p}) \tag{42}$$

are finite at the point $x=y$. Indeed, for example, the function $D_c^R(0)$ in the Euclidean metric is given by

$$\begin{aligned}
D_c^R(0)&=\frac{\pi^2}{(2\pi)^4}\int_0^\infty \frac{du\cdot u}{m^2+u}\cosh^{-2}[l(m^2+u)^{1/2}]\\
&=\frac{1}{16\pi^2}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\frac{v(\rho)}{\sin\pi\rho}\frac{m^{2+\rho}}{\Gamma(1+\rho)}l^\rho\frac{\Gamma(-1-\frac{1}{2}\rho)}{\Gamma(1-\frac{1}{2}\rho)}\\
&=\frac{m^2}{4\pi^2}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\frac{v(\rho)}{\sin\pi\rho}\frac{(ml)^\rho}{\Gamma(1+\rho)}\frac{1}{\rho(2+\rho)}\\
&=\frac{1}{8}\frac{m^2}{\pi^2}\left[\frac{\ln 2}{m^2l^2}+v'(0)+\ln ml-\psi(1)-\frac{1}{2}\right]\qquad (3<\gamma<2)
\end{aligned}$$

Similarly to the formula (30) the following Mellin representation holds for the functions (39) and (40):

$$\tilde{D}_c^R(p)=\frac{1}{2i}\int_{\substack{-\gamma+i\infty\\(2<\gamma<1)}}^{-\gamma-i\infty} d\rho\frac{v(\rho)}{\sin\pi\rho\Gamma(1+\rho)}l^\rho(m^2-p^2-i\varepsilon)^{(1/2)\rho-1} \tag{43}$$

and

$$\tilde{S}_c^R(\hat{p})=(m+\hat{p})\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\frac{v_1(\rho)}{\sin\pi\rho\Gamma(1+\rho)}l^\rho(m^2-p^2-i\varepsilon)^{(1/2)\rho-1} \tag{44}$$

respectively. Here $v(\rho)$ is given by the formula (29). Now we define the explicit form of $v_1(\rho)$ by means of the Mellin representation for $\cosh^{-1}x$. Since both the following expansions are valid for $\cosh^{-1}z$:

$$\cosh^{-1}z=2\sum_{k=1}^\infty(-1)^k\exp[-(2k+1)z]\qquad z>0$$

and

$$\cosh^{-1}z=\sum_{n=0}^\infty E_n\frac{z^n}{n!}\qquad \text{for } z^2<\tfrac{1}{4}\pi^2$$

Further, we have

$$\cosh^{-1} z = \frac{1}{2i} \int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho \frac{v_1(\rho) z^\rho}{\sin \pi\rho \Gamma(1+\rho)}, \qquad (2<\gamma<1) \tag{45}$$

where

$$v_1(\rho) = \begin{cases} 2\beta(-\rho) & \text{for } \rho<0 \\ E_\rho & \text{for } \rho \geq 0 \end{cases}$$

Here

$$\beta(\rho) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k+1)^\rho}, \qquad \beta(2n+1) = \frac{\pi^{2n+1}}{2^{2n+2}} \frac{E_n}{(2n)!}$$

E_n are Euler numbers, $E_{2n+1}=0$; $n=0, 1, 2, \ldots$; in particular

$$\begin{aligned} E_0 &= 1, & E_6 &= -61 \\ E_2 &= -1, & E_8 &= 1385 \\ E_4 &= 5, & E_{10} &= -50521 \end{aligned}$$

As in the case of the Coulomb law, in the given scheme the Yukawa potential between two scalar particles acquires the form

$$\begin{aligned} U_Y(r) &= \frac{g^2}{(2\pi)^3} \int d^3p \frac{e^{i\mathbf{pr}}}{m^2+\mathbf{p}^2} \cosh^{-2}[l(m^2+\mathbf{p}^2)^{1/2}] \\ &= \frac{g^2}{2\pi^2 r} \frac{m}{\sqrt{\pi}} \frac{1}{2i} \int_{\substack{-\gamma+i\infty \\ (4<\gamma<3)}}^{-\gamma-i\infty} d\rho \frac{v(\rho) l^\rho}{\sin \pi\rho \Gamma(1+\rho)} (2m/r)^{(1/2)(\rho-1)} \\ &\quad \times \cos \tfrac{1}{2}\pi(\rho-1) \cdot \Gamma(\tfrac{1}{2}\rho) K_{(1/2)(\rho+1)}(mr) \end{aligned} \tag{46}$$

This representation is valid for the case $r=0$ and has only a single pole at the point $\rho=0$. The calculation of the residue at this point gives

$$U_Y(r) = \frac{g^2}{4\pi r} e^{-mr} \qquad \text{for } r \neq 0 \tag{47}$$

In order to calculate the value of $U_Y(r)$ for $r \to 0$, we use another representation obtained directly from the first equality in (46):

$$\lim_{r\to 0} U_Y(r) = U_Y(0) + r^2 U_Y^1 + O(r^4)$$

where

$$U_Y(0)=\frac{g^2}{2\pi^2}\int_0^\infty \frac{dx\,x^2}{m^2+x^2}\cosh^{-2}[l(m^2+x^2)^{1/2}]$$
$$=\frac{g^2}{2\pi^2}\frac{m}{2}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\frac{v(\rho)}{\sin\pi\rho}\frac{(ml)^\rho}{\Gamma(1+\rho)}\frac{\Gamma(\frac{3}{2})\Gamma(-\frac{1}{2}-\frac{1}{2}\rho)}{\Gamma(1-\frac{1}{2}\rho)}$$
$$=-\frac{g^2}{2\pi^2 l}-\frac{mg^2}{4\pi}-\frac{7}{4}m\frac{g^2}{\pi^4}(ml)\zeta(3)+O(m^2l^2) \tag{48a}$$

$$U_Y^1=-\frac{g^2}{2\pi^2}\frac{1}{3!}\int_0^\infty\frac{dx\,x^4}{m^2+x^2}\cosh^{-2}[l(m^2+x^2)^{1/2}]$$
$$=-\frac{g^2}{2\pi^2}\frac{1}{3!}\frac{m^3}{2}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\frac{v(\rho)(ml)^\rho}{\sin\pi\rho\Gamma(1+\rho)}\frac{\Gamma(\frac{5}{2})\Gamma(-\frac{3}{2}-\frac{1}{2}\rho)}{\Gamma(1-\frac{1}{2}\rho)}$$
$$=-\frac{g^2}{144l^3}-\frac{g^2m^2}{8\pi^2 l}-\frac{g^2m^3}{24\pi}+O(ml) \tag{48b}$$

Here $\zeta(3)=\sum_{n=1}^\infty 1/n^3=1.202\,056\,90$.

Unifying the formulas (47), (48a), and (48b), we have

$$U_Y(r)=\begin{cases}-\dfrac{g^2}{2\pi^2 l}-\dfrac{m}{4\pi}g^2-\dfrac{7mg^2}{4}\dfrac{ml}{\pi^4}\zeta(3)+O(m^3l^3) & \\ \qquad -r^2\left[\dfrac{g^2}{144l^3}+N\right] & \text{for } r\to 0\\ (g^2/4\pi r)\,e^{-mr} & \text{for } r\neq 0\end{cases} \tag{49}$$

where $N=(g^2/8\pi^2)(m^2/l)+(g^2/24\pi)m^3+O(ml)$. Thus, we see that the Yukawa law is valid up to the point $r=0$ and, therefore, the corresponding theory is local almost everywhere.

Now we go over to a discussion of the equation (37). As in the massless case it has three solutions:

$$\begin{array}{lll}(1)\quad p^2=m^2, & \text{at which } \cosh 0=1 & \\ (2)\quad p^2\neq m^2, & \cos[l(p^2-m^2)^{1/2}]=0, & p^2>m^2\\ (3)\quad \text{trivial solution } \tilde{\varphi}_R(p)\equiv 0, & \text{for } m^2>p^2 & \end{array} \tag{50}$$

The former corresponds first to the free scalar particle with mass m and second to the family of particles with masses

$$M_n=\{m^2+[(\pi/l)(\tfrac{1}{2}+n)]^2\}^{1/2},\qquad n=0,1,2,\ldots \tag{51}$$

In the second case the initial particle becomes a virtual one but at the same time a family of particles is generated due to the quantized space-time properties at small distances. On the other hand, these new generated particles may be understood as excited states of the initial particle with discrete energy levels

$$E_n = (E_0^2 + E_n'^2)^{1/2} \tag{52}$$

in quantized space-time, where

$$E_0 = (m^2 + \mathbf{p}^2)^{1/2} \qquad \text{and} \qquad E_n' = (\pi/l)(\tfrac{1}{2} + n)$$

4. FINITE QUANTUM ELECTRODYNAMICS IN QUANTIZED SPACE-TIME

The aim of this section is to construct a finite QED within the framework of quantized space-time. As seen above, our hypothesis leads to a change of the particle propagators at large scale. This change is the same as in the case of nonlocal-stochastic theory considered in the references (Efimov, 1977; Namsrai, 1985) and, therefore, we use here their mathematical methods.

4.1. The Construction of the S matrix

The Lagrangian of a system of fields is constructed by means of the extended fields $A_\mu^R(x)$ and $\psi_R(x)$ of photon and leptons in the Minkowski space. As usual, the initial Lagrangian describing the electromagnetic interactions of leptons is chosen in the form

$$\mathcal{L}(x) = \mathcal{L}_0 + \mathcal{L}_{\text{em}}(x)$$

$$\mathcal{L}_0 = -\tfrac{1}{2}\colon [\hat{\mathcal{P}}_\nu A_\mu^R(x)]^2 \colon + \sum_j \colon \bar{\psi}_j^R(x)(i\hat{\partial} - m_j)$$

$$\times \cosh^{-1}[l(m^2 - \Box)^{1/2}]\psi_j^R(x)\colon \tag{53}$$

where $A_\mu^R(x)$ and $\psi_j^R(x)$ are the extended fields of photon and leptons ($j = e, \mu$). The subindex R will henceforth be omitted. As in the case of nonlocal-stochastic theory, the S matrix can be written formally in the form of the T products:

$$S = 1 + i\sum_{n=1}^{\infty} \frac{1}{n!} S_n$$

$$S_n = i^{n-1} \int d^4x_1 \cdots \int d^4x_n T_d \left\{ \prod_{j=1}^{n} \mathcal{L}_{\text{em}}(x_j) \right\} \tag{54}$$

Here the symbol T_d refers to the so-called Wick T product or T^* operation and the lower case d corresponds to the algebraic presciption determined in Section 4.2 which follows.

In order to study the perturbation series for the S matrix (54) in our scheme it is necessary to use prescriptions of the usual local theory and to change (in the Feynman diagrams)

$$(m+\hat{k})(m^2-k^2-i\varepsilon)^{-1} \Rightarrow (m+\hat{k})(m^2-k^2-i\varepsilon)^{-1}\cosh^{-1}[l(m^2-k^2)^{1/2}]$$
$$g_{\mu\nu}(-k^2-i\varepsilon)^{-1} \Rightarrow g_{\mu\nu}(-k^2-i\varepsilon)^{-1}\cosh^{-2}[l(-k^2)^{1/2}]$$

at the same time inserting the modified function

$$\gamma_\mu \to U_\mu(q,k) = -d_\mu(k)S_R^{-1}(\hat{q})$$

into the vertices at the external photon lines. The calculations of the matrix elements for the charged lepton loops will be carried out using the methods of the nonlocal-stochastic theory (Namsrai and Dineykhan, 1983) (see, also below).

4.2. The Problem of Gauge Invariance in the Given Scheme

It is well known that a change of the propagator of leptons leads to a violation of some algebraical relations (e.g., the Ward–Takahashi identities), the fulfilment of which grants the gauge invariance of the theory. This problem has been discussed in detail by some authors (see, for example, Namsrai and Dineykhan, 1983) and therefore we do not repeat their results and pause only at the main issues:

1. The form of the one-photon vertex should be changed,

$$\gamma_\mu \to U_\mu(q,k) = -d_\mu(k)S_R^{-1}(\hat{q}) \tag{55}$$

according to the Ward–Takahashi identity

$$k_\mu \Gamma_\mu(p,k) = S_R(\hat{p}) - S_R(\hat{q}) \tag{56}$$

where

$$\Gamma_\mu(p,k) = S_R(\hat{p})U_\mu(k,q)S_R(\hat{q}) \qquad (p=k+q)$$

Here $d_\mu(k)$ is some operator whose action on the function $V(-q^2l^2)$ is given by

$$d_\mu(k)V(-q^2l^2) = [V(-(q+k)^2l^2) - V(-q^2l^2)]\frac{\hat{k}\gamma_\mu}{k^2} \tag{57}$$

In our case $V(x)$ equals either $\cosh^{-2} x$ or $\cosh^{-1} x$.

2. Interactions of n photons with open charged propagators is given by the following formula:

$$S_R(\hat{q}_n)\Gamma_n(q; k_1, \dots, k_n)S_R(\hat{q}) = (-1)^n d(k_1) \cdots d(k_n) S_R(\hat{q}) \tag{58}$$

where $q_n = q + \sum_{i=1}^{n} k_i$, in particular

$$\Gamma_{1\mu}(k, q) = U_{1\mu}(k, q) = -d_\mu(k) S_R^{-1}(\hat{q})$$

and

$$d_\mu(k) S_R(\hat{q}) = S_R(\hat{q} + \hat{k})\Gamma_{1\mu}(k, q) S_R(\hat{q}) \tag{59}$$

It is easy to verify that the generalized Ward–Takahashi identity

$$(p_\mu - q_\mu)\Gamma_{1\mu}(p, q) = S_R(\hat{p}) - S_R(\hat{q}) \tag{60}$$

is valid in accordance with the formulas (57) and (59).

3. In our scheme the charged loop is determined by the following expression:

$$\Pi_n^R(k_1, \dots, k_n) = \frac{1}{n} \int d^4 q \,\mathrm{Sp}\{\Gamma_n^R(q; k_1, \dots, k_n) S_R(\hat{q})\} \tag{61}$$

where

$$\Gamma_n^R(q; k_1, \dots, k_n) = V(-q_n^2 l^2)\Gamma_n(q; k_1, \dots, k_n)$$

and

$$S_R(\hat{q}_n)\Gamma_n(q; k_1, \dots, k_n) S_R(\hat{q})$$

is given by (58).

Notice that these obtained relations imply gauge invariance for the S matrix in any order of the perturbation series (for detail, see Namsrai and Dineykhan, 1983).

4.3. Study of the Perturbation Theory

Recall that our form factor $V(-q^2 l^2)$ oscillates in the physical region of $q^2 \to +\infty$ (i.e., in the Minkowski space) with the exception of isolated points $q_n^2 > 0$ at which it may turn to infinity. On the other hand the function $V(-q^2 l^2)$ decreases only in the Euclidean direction, i.e., when $q^2 \to -\infty$. Therefore we shall investigate the Feynman diagrams in the Euclidean momentum space. In our model, some intermediate regularization procedure may be used in order to open a passage to the Euclidean metrics; however, it is not as necessary as in the case of nonlocal-stochastic theory, where the form factors $V(-q^2 l^2)$ increase rapidly enough, at least as the linear exponent $\exp[l(p^2)^{1/2}]$, when $p^2 \to +\infty$.

Since, in our scheme the above-mentioned infinities are situated on the real axis of Re q_0 and their presence does not affect the display of the counter integrations in Feynman diagrams to the Euclidean region $q_0 \to iq_4$ (see Figure 2), therefore, we do not use here an intermediate regularization procedure.

Let us calculate the matrix elements for the S matrix corresponding to the following primitive diagrams (see Figure 3) which are divergent in the usual quantum electrodynamics.

4.3.1. Vacuum Polarization Diagrams

In the gauge-invariant QED constructed by means of the hypothesis of quantized space-time, the vacuum polarization in the second order of perturbation theory is determined by diagrams sketched in Figure 3a.

In the momentum representation the term of the S matrix which corresponds to these diagrams is given by an expression of the type (61)

$$\Pi^R_{\mu\nu}(k_1, k_2) = \frac{ie^2}{(2\pi)^4} \frac{1}{2} \int d^4q V_1(-q^2l^2)\, \mathrm{Sp}\{\Gamma_{\mu\nu}(q; k_1, k_2) S_R(\hat{q})\}$$

$$(k_1 + k_2 = 0) \quad (62)$$

where

$$S(\hat{q}_2)\Gamma_{\mu\nu}(q; k_1, k_2)S(\hat{q}) = (-1)^2 d_\mu(k_1) d_\nu(k_2) S(\hat{q})$$

and

$$V_1(-q^2l^2) = \cosh^{-1}[l(m^2 - q^2)^{1/2}], \qquad q_2 = q + k_1 + k_2 = q,\ S = S_R$$

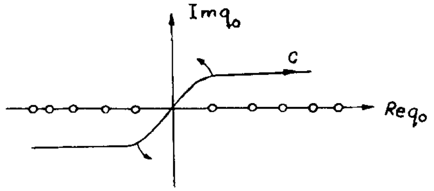

Fig. 2. The form of counter integrations in Feyman diagrams in our model.

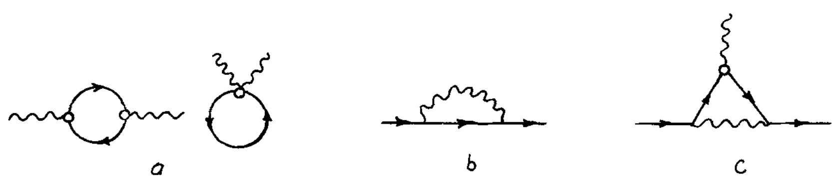

Fig. 3. The primitive divergent diagrams of QED constructed by means of the idea of quantized space-time.

The last function is determined by the Mellin representation (45) [for the causal Green's function $S(\hat{q})$ see the formula (44)]. Equation (62) is simplified by the d operation determined above. As usual, taking the trace, integrating over d^4q and going to the Euclidean metric we obtain the gauge invariant quantity:

$$\Pi^R_{\mu\nu}(k) = \frac{e^2}{2\pi^2}(k_\mu k_\nu - g_{\mu\nu}k^2)\Pi(k^2)$$

$$\Pi(k^2) = \frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho \frac{k_1(\rho)}{\sin \pi\rho}(m^2l^2)^\rho \int_0^1 dx\, x(1-x)^{1-\rho}\frac{\Gamma(-\rho)}{\Gamma(1-\rho)}\mathscr{L}_0^\rho$$

where $\mathscr{L}_0 = 1-(k^2/m^2)x(1-x)$.

Assuming $m^2l^2 \ll 1$ we get

$$\Pi(k^2) = \int_0^1 dx\, x(1-x)\left\{\ln[\mathscr{L}_0(x)/x(1-x)] - \frac{5}{6} + k_1'(0) + \ln m^2l^2 + O(m^2l^2)\right\}$$

where

$$k_1(x) = \frac{v_1(2x)}{\cos \pi x\Gamma(1+2x)}$$

We see that after the charge renormalization the value obtained for the vacuum polarization coincides with the renormalized expression in the usual local theory (see, for example, Bogolubov and Shirkov, 1980).

4.3.2. *The Diagram of Self-Energy (Fig. 3b)*

The amplitude for the proper self-energy may be expressed as

$$-i{:}\; \bar{\psi}_R(x)\Sigma_R(x-y)\psi_R(y){:}$$

where

$$\Sigma_R(x-y) = (2\pi)^{-4}\int d^4p\, e^{ip(x-y)}\tilde{\Sigma}_R(p)$$

Here

$$\tilde{\Sigma}_R(p) = \frac{-ie^2}{(2\pi)^4}\int d^4k \frac{V(-k^2l^2)}{-k^2-i\varepsilon}\gamma_\mu \frac{m+\hat{p}-\hat{k}}{m^2-(p-k)^2-i\varepsilon}\gamma_\mu V_1(-(p-k)^2l^2)$$

$V(-k^2l^2) = \cosh^{-2}[l(m^2-k^2)^{1/2}]$, and $V_1(x)$ as in (62). In order to calculate explicitly this integral we can use the representations (28) and (45) for the form factors $V(x)$ and $V_1(x)$, respectively, and also the general Feynman parametric form:

$$b_1^{-\mu_1}\cdots b_n^{-\mu_n}=\frac{\Gamma(\mu_1+\cdots+\mu_n)}{\Gamma(\mu_1)\cdots\Gamma(\mu_n)}\int_0^1 dx_1\cdots\int_0^1 dx_n\,\delta\left(1-\sum_{j=1}^n x_j\right)$$
$$\times x_1^{\mu_1-1}\cdots x_n^{\mu_n-1}\left[\sum_{j=1}^n x_j b_j\right]^{-\mu_1-\cdots-\mu_n}$$

Then, after some standard calculations, we get

$$\tilde{\Sigma}_R(p)=\frac{e^2}{8\pi^2}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho\,\frac{k(\rho)}{\sin\pi\rho}(m^2l^2)^\rho$$
$$\times\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta\,\frac{k_1(\eta)}{\sin\pi\eta}(m^2l^2)^\eta$$
$$\times\frac{\Gamma(-\eta-\rho)}{\Gamma(1-\eta)\Gamma(1-\rho)}\int_0^1 dx\left(\frac{1-x}{x}\right)^\rho\left(1-\frac{p^2}{m^2}x\right)^{\rho+\eta}(2m-\hat{p}x) \tag{63}$$

Assuming the value of m^2l^2 to be small and displaying integration counter to the right, one can obtain for the self-energy the following expressions:

$$\tilde{\Sigma}_R(p)=\frac{e^2}{8\pi^2}\int_0^1 dx\,(2m-\hat{p}x)\ln\frac{m^2}{m^2-p^2x}$$
$$+\frac{e^2m}{16\pi^2}\left\{[3\ln(m^{-2}l^{-2})-3k'(0)-1\right.$$
$$\left.+\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta\,\frac{k_1(\eta)k(-\eta)}{\sin^2\pi\eta}(3-\eta)\right]$$
$$\left.+O(m^2l^2)\right\}+\frac{e^2}{16\pi^2}(m-\hat{p})\left\{\left[\ln(m^{-2}l^{-2})-k'(0)+1\right.\right.$$
$$\left.\left.+\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta\,\frac{k_1(\eta)k(-\eta)}{\sin^2\pi\eta}(1-\eta)\right]+O(m^2l^2)\right\} \tag{64}$$

Similarly to the function $k_1(x)$ presented above the function $k(x)$ in (63) and (64) is given by

$$k(x)=\frac{v(2x)}{\cos\pi x\Gamma(1+2x)}$$

The calculation of integrals of the type

$$\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta\,\frac{k_1(\eta)k(-\eta)}{\sin^2\pi\eta}(\cdots)$$

may be made if it is necessary. Thus, we see that the value obtained for the self-energy differs slightly from the value calculated in the stochastic-nonlocal theory (Efimov, 1972; Namsrai and Dineykhan, 1983).

4.3.3. The Vertex Diagram and the Correction to the Anomalous Magnetic Moment of the Leptons

Now we turn to the study of the vertex diagram shown in Figure 3c. In the momentum representation it has the standard form

$$\tilde{\Gamma}^R_\mu(p, q) = \frac{e^2}{(2\pi)^4 i}\int d^4k\, D_R(-(p-k)^2 l^2)\gamma_\nu d_\mu(q) S(\hat{k})\gamma_\nu$$

where in accordance with d operation

$$d_\mu(q)S(\hat{k}) = \frac{1}{m-\hat{k}-\hat{q}}\gamma_\mu \frac{V_1(-k^2l^2)}{m-\hat{k}} - \frac{1}{m-\hat{k}-\hat{q}} \times\left[V_1(-(k+q)^2l^2) - V_1(-k^2l^2)\right]\frac{\hat{q}\gamma_\mu}{q^2}$$

By using the identity

$$a^n - b^n = n(a-b)\int_0^1 dx\,[(a-b)x+b]^{n-1} \tag{65}$$

the difference of the form factor values can be transformed to the convenient form for concrete calculations

$$V_1(-(k+q)^2l^2) - V_1(-k^2l^2) = -[q^2 + 2(k\cdot q)]\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta \frac{k_1(\eta)}{\sin \pi\eta} l^{2\eta} \times \eta\int_0^1 dx[m^2 - k^2 - 2x(k\cdot q) - q^2x]^{\eta-1}$$

It is standard procedure to express $\tilde{\Gamma}^R_\mu(p, q)$ for the matrix element of the vertex functions between two free single-lepton states in the form

$$\tilde{\Gamma}^R_\mu(p, q) = \bar{u}_j(p')\left[\gamma_\mu F_1(q^2) + \frac{i}{2m_j}\sigma_{\mu\nu}q^\nu F_2(q^2)\right]u_j(p) \tag{66}$$

where $u(p)$ and $\bar{u}(p)$ are the Dirac spinors and

$$F_1(q^2) = f_1(q^2) + f_2(q^2), \qquad F_2(q^2) = g_1(q^2) + g_2(q^2) \qquad (j = e, \mu)$$

Here

$$\sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

$$f_1(q^2) = N(\rho, \eta)\left\{\left[-2+8\beta-2\beta^2-2\frac{g^2}{m_j^2}(1-\gamma)(1-\alpha)\right]\right.$$
$$\left.\times\Gamma(1-\eta-\rho)\mathscr{L}_1^{-1+\eta+\rho} - 2\Gamma(-\eta-\rho)\mathscr{L}_1^{\eta+\rho}\right\}$$

$$g_1(q^2) = 4N(\rho, \eta)\beta(1-\beta)\Gamma(1-\eta-\rho)\mathscr{L}_1^{-1+\eta+\rho}$$

$$f_2(q^2) = N(\rho, \eta)\eta\int_0^1 dt\left[2\frac{q^2}{m_j^2}(1-\alpha-t\gamma)(1-\beta-2\alpha-2t\gamma)\right.$$
$$\left.\times\Gamma(1-\eta-\rho)\mathscr{L}_2^{-1+\eta+\rho} - 2\Gamma(-\eta-\rho)\mathscr{L}_2^{\eta+\rho}\right]$$

$$g_2(q^2) = 4N(\rho, \eta)\eta\int_0^1 dt\beta[1-\beta-2\alpha-2t\gamma]\Gamma(1-\eta-\rho)\mathscr{L}_2^{-1+\eta+\rho}$$

$$N(\rho, \eta) = \frac{e^2}{(2\pi)^4}\frac{\pi^2}{2i}\int_{-\delta+i\infty}^{-\delta-i\infty} d\rho\frac{k(\rho)(m_j^2l^2)^\rho}{\sin\pi\rho}\frac{1}{2i}\int_{-\zeta+i\infty}^{-\zeta-i\infty} d\eta\frac{k_1(\eta)}{\sin\pi\eta}(m_j^2l^2)^\eta$$
$$\times\frac{1}{\Gamma(1-\rho)\Gamma(1-\eta)}\iiint_0^1 d\alpha\, d\beta\, d\gamma\cdot\beta^{-\rho}\gamma^{-\eta}\delta(1-\alpha-\beta-\gamma)$$

$$\mathscr{L}_1(q^2) = (1-\beta)^2 - \frac{q^2}{m_j^2}\alpha\gamma \qquad (m_j = m_e, m_\mu)$$

$$\mathscr{L}_2(q^2) = \mathscr{L}_1(q^2) - \frac{q^2}{m_j^2}t\gamma(1-t\gamma) + 2\frac{q^2}{m_j^2}t\alpha\gamma + \frac{q^2}{m_j^2}t\gamma\beta$$

As a result of calculations of the counter integrals the first term of (66), i.e., $F_1(q^2)$ in the limit $q^2 \to 0$ and with the assumption $m_j^2l^2 \ll 1$ takes the form

$$F_1(q^2) = -\frac{\alpha}{4\pi}\left\{\ln(m_j^{-2}l^{-2}) - 2\ln(m_j^2/m_\gamma^2) - k'(0) + \frac{9}{4} - m_j^2l^2k(1)\right.$$
$$\left.\times\left[-\frac{4}{3}k'(0) + 4\frac{k'(1)}{k(1)} - \frac{8}{3}\ln(m_j^{-2}l^{-2}) - \frac{20}{9}\right]\right\}$$
$$-\frac{\alpha}{2\pi}\frac{q^2}{m_j^2}\left\{\frac{2}{3}\left(\ln\frac{m_j}{m_\gamma} - \frac{5}{8}\right)\right.$$
$$\left.+k(1)m_j^2l^2\left[\frac{1}{3}k'(0) + \frac{2}{3}\frac{k'(1)}{k(1)} + \ln m_j^2l^2 - \frac{3}{4}\right]\right\}, \qquad \alpha = \frac{e^2}{4\pi}$$

and contains the terms corresponding to the charge renormalization of the leptons. Here m_γ is the photon mass which has arisen from the removal procedure of infrared infinities. It is well known that the second term of (66) at $q^2=0$ contributes to the the anomalous magnetic moment of the leptons by

$$a_j = F_2(0) = \frac{4}{(2i)^2}\frac{\alpha}{2\pi}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta \frac{k_1(\eta)}{\sin \pi\eta}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\rho \frac{k(\rho)}{\sin \pi\rho}(m_j^2 l^2)^{\eta+\rho}$$
$$\times \frac{\Gamma(1-\eta-\rho)\Gamma(1+2\rho+\eta)}{\Gamma(3-\eta)\Gamma(3+\eta+\rho)}(1-\eta)(1-\rho)$$

Assuming $m_j^2 l^2 \ll 1$ ($m_j = m_e, m_\mu$) we obtain

$$a_j \simeq \frac{\alpha}{2\pi}\left\{1 + m_j^2 l^2\left[-\frac{2}{3}k(1) - \frac{2}{3}\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta \frac{k_1(\eta)k(1-\eta)}{\sin^2 \pi\eta}\eta(1-\eta)\right]\right\}$$
$$\simeq \frac{\alpha}{2\pi}\left[1 - \frac{2}{3}m_j^2 l^2 k(1)\right] \tag{67}$$

Recently, the experimental values of lepton AMM (Van Dyck et al., 1979; Bailey et al., 1979)

$$a_{\text{exp}}(e^-) = (1\,159\,652\,200 \pm 40)\times 10^{-12}, \qquad a_{\text{exp}}(\mu) = (1\,165\,924 \pm 8.5)\times 10^{-9}$$

and their theoretical calculations (Calmet et al., 1977; Kinoshita, 1979; Schwinberg et al., 1981) due to local QED coincide with perfect accuracy, for example,

$$a_{\text{theor}}(e) = (1\,159\,652\,411 \pm 166)\times 10^{-12}$$
$$= \frac{\alpha}{2\pi} - 0.328\,478\,455\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4$$

where $C_3 = 1.1765 \pm 0.0013$, $C_4 = -0.8 \pm 2.5$,

$$\alpha^{-1} = 137.035\,963 \pm 0.000\,015$$

From this fact it is natural to suppose that the contribution calculated here should be of an order no greater than the experimental errors. This makes it possible to establish the following restrictions on the parameter l:

$$\begin{aligned} l &\lesssim 8.9\times 10^{-15}\ \text{cm} \qquad (e^-) \\ l &\lesssim 6.3\times 10^{-16}\ \text{cm} \qquad (\mu) \end{aligned} \tag{68}$$

Stricter estimations for the value of l can be obtained by using experimental data on the scattering of electrons at high energies. Since electromagnetic processes of the type $e^-e^- \to e^-e^-$, $e^+e^- \to e^+e^-$ and $e^+e^- \to \mu^+\mu^-$ are described even by a low order of the perturbation theory up to, at high energies, the recently attainable one (see Figure 4), the ratio of cross sections calculated by the usual local and nonlocal theory discussed above will be given by the following formula:

$$\sigma_{\text{nonloc}}/\sigma_{\text{loc}} = [V(-sl^2)]^2 \simeq [1 - \tfrac{1}{2}v(2)sl^2]^2 \simeq 1 - 2sl^2$$

where $v(2)$ is given by the expression (29), and $s = (p_1 + p_2)^2 = (2E)^2 = W^2$, $W = 2E$ is total energy in the center of the mass frame of reference.

An estimation based on this formula is very simple and using present experimental data (see, for example, Bartel et al., 1980 and Berger et al., 1980) we have

$$l \lesssim 10^{-16}\,\text{cm} \tag{69}$$

Of course, there exist other bounds on the fundamental length (see, for example, Bracci et al., 1983).

Thus, summarizing both the estimations (68) and (69) one can say that the local QED is excellently described experimental data up to distances of the order of 10^{-15}–10^{-16} cm. However, it should be noted that at very high energies the testing of locality of QED, i.e., obtaining the estimation (69) may be difficult because of interference effects between the electromagnetic and weak interactions. For example, in the standard model of electroweak interactions, the testing of QED will be disturbed by interference with the weak effects due to Z^0 bosons.

To conclude this section we notice that the S matrix obtained is gauge invariant. Indeed, in the modified quantum electrodynamics under consideration the Ward identity

$$\frac{\partial}{\partial p_\mu}\tilde{\Sigma}_R(p) = -\tilde{\Gamma}^R_\mu(p, 0)$$

is valid because this identity is a direct consequence of the identities (55) and (59). Since we must not make any subtractions of infinite counterterms,

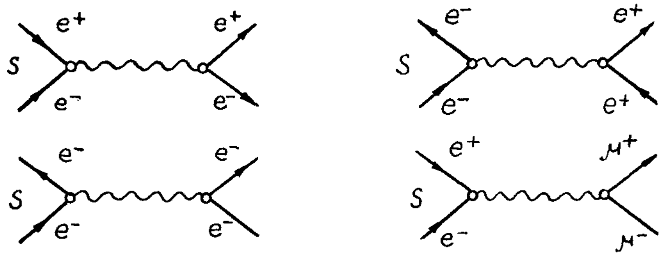

Fig. 4. Diagrams of electromagnetic leptonic processes giving the main contribution to the scattering value at high energies.

no dangerous terms which can break the Ward identity when the formula (59) is valid will appear in the perturbation theory. The diagram of the vacuum polarization is gauge invariant due to our choice of the generalized gauge-invariant regularization procedure originated by Kroll (1966).

5. SOME SPECULATION AND DISCUSSION

Thus, we have here a given concrete form of quantized space-time within which we have attempted to construct a quantum field theory, for example, of quantum electrodynamics. It transpired that the idea of quantized space-time at small distances leads to a change of the momentum operator $\hat{\mathscr{P}}_\mu$ at large scale. In turn the latter makes interaction between fields nonlocal, the character of which is revealed in quantum field theory as a residual effect (the change of particle propagators) resulting from the averaging procedure at small distances. Moreover, the scheme discussed here may be useful for further study, from the indication that it contains, from the physical point of view, the following interesting possibilities:

1. This model may provide some way to describe tachyon-type objects. Indeed, from equation (37) we see that if $m \to iM$ is inserted into it, we obtain

$$(p^2+M^2)\cos^2[l(M^2+p^2)^{1/2}]\tilde{\varphi}_R(p)=0 \tag{70}$$

which has a family of solutions

$$M_n=(p_n^2)^{1/2}=\{[(\pi/l)(\tfrac{1}{2}+n)]^2-M^2\}^{1/2} \tag{71}$$

It means that the very existence of tachyon-type objects gives rise to the appearance of a family of physical particles in quantized space-time, if $(\pi/2l)\geq M$ at the least. In other words, tachyons may be understood equivalently as a family of physical particles with quantized masses given by the formula (71) and from this it seems conceivable that tachyons may not exist independently as free particles.

In accordance with the restriction (69) we have the following estimation for the mass value of tachyons if they exist:

$$M\leq(\pi/2l)\sim 157\ \text{GeV}$$

2. It is quite possible that our scheme may open a way to the understanding of equidistance behavior of observed particle mass spectra or, in more general cases it may provide a rule of particle mass quantization, since in our model there exist particles, masses of which are quantized by the formulas (18) and (51):

$$M_n=\frac{\pi}{l}\left(\frac{1}{2}+n\right) \qquad \text{or} \qquad M_n'=[m_0^2+(\pi^2/l^2)(\tfrac{1}{2}+n)^2]^{1/2}$$

This fact prompts us to consider the quantization of elementary particle masses from the quantized space-time point of view. However, of course, our simpler method of introducing quantized space-time [see the formula (1)] does not describe observable particle mass spectra. Moreover, in this case, the whole problem depends on how to understand the value of l—the parameter of the theory. If it is assumed that parameter l loses its universal character due to interference effects caused by extended objects (quantized elementary domains) such as bags, strings, monopoles, different compound states of quarks and gluons, etc. (called test particles) then the value of l may be understood as an effective size (or size of quantized domains) $l \to l_{\text{eff}}$ of the test particles. For example, in accordance with this ideology, the value of l discussed above in quantum electrodynamics may now be understood as an electron size and from the estimate (69) it may be said that the electron is a pointlike particle (i.e., structureless) down to a distance of the order of 10^{-16} cm.

Thus, assuming $l \to l_{\text{eff}} \sim 5 \times 10^{-15}$ cm and putting this into (18) we obtain the following family of particles, massive photons:

$$
\begin{array}{lll}
M_0 = 6.28\ \text{GeV}, & M_3 = 44\ \text{GeV}, & M_6 = 81.64\ \text{GeV} \\
M_1 = 18.84\ \text{GeV}, & M_4 = 56.52\ \text{GeV}, & M_7 = 94.2\ \text{GeV} \\
M_2 = 31.4\ \text{GeV}, & M_5 = 69.1\ \text{GeV}, & M_8 = 106.76\ \text{GeV} \\
 & & \cdots\cdots\cdots\cdots
\end{array}
$$

We see that the sixth and seventh particles with the masses M_6 and M_7 from this family concide with $W^{\pm}$ and Z^0 bosons of the electroweak interactions. Other particles $M_0 - M_5$ or $M_8 - \cdots$ may exist as bounded (or excited) states of quarks and gluons, i.e., gluonlike particles with masses. Analogously, one can construct a scalar($\pi, K, \ldots$), vector($\rho, \omega, \ldots$), and spinor($p, e, \ldots$)-like family of particles by the formula (51). At this the value of l_{eff} may be chosen in a different manner.

It should be noted that another possibility may exist: The parameter l may indeed be, as we wished, a universal fundamental constant, and its value may be very small, say $l \sim 10^{-33}$ cm. In this case, the above-mentioned proposal does not work and the family of particles does not exist; but instead of these there may exist super-heavy particles with quantized masses of the order of the Planck mass $M_{\text{Pl}} \sim 10^{-5}$ g (or $M_{\text{Pl}} = 10^{19}$ GeV), and their energy takes the discrete values:

$$E_{\text{Pl}} = M_{\text{Pl}} c^2 \pi (\tfrac{1}{2} + n), \qquad n = 0, 1, 2, \ldots \tag{72}$$

This formula is obtained from (52) by the assumption $m \ll M_{\text{Pl}} \sim 1/l$ and $\mathbf{p}^2 = 0$. A similar problem has been discussed in quantum gravitational theory (see, for example, Markov, 1984).

3. As is pointed out, the hypothesis of quantized space-time leads to a change of the interaction law between two bodies at small distances. The essence of this change is very important in explaining the nature of the force which characterizes the behavior of particles, and their bounded states and ratio of intensities of different types of forces: weak, strong, and electromagnetic interactions at small distances. Moreover, our scheme may give a key to understanding the problem of quark confinement which, it seems, is caused by the character of force of the type $F(r) \sim gr$ or $g \ln r$, where g is the coupling constant. It turns out that within the framework of our model one can realize this type of force between two bodies. Indeed, from expressions (32) and (49) it is clear that

$$F_{\mathrm{EM}}^{\mathrm{attr}} = -\frac{\partial U_C(r)}{\partial r} = \frac{e^2}{72l^3} r, \qquad e^2/4\pi = 1/137$$

and

$$F_{\mathrm{ST}}^{\mathrm{attr}} = -\frac{\partial U_Y(r)}{\partial r} = \frac{g^2}{72l^3} r, \qquad g^2/4\pi = 15 \qquad \text{for } r \lesssim l \tag{73}$$

This means that at very small distances $r \lesssim l$, the character of the Coulomb and Yukawa laws is changed and acquires the attractive form. It is quite possible that that character gives rise to the appearance of confinement in nature. In this connection it should be noted that there appears the interesting possibility that electrical charges of the same (different) name are attracted (repulsed) at small distances. This effect of course promotes the formation of bounded states of quarks with different values of electric charges of the same name. Roughly speaking, this effect is connected with the situation that when two charges of the same name are built up in a bag of radius l then their further separation from each other is indeed impossible owing to the attractive nature of the force between them.

Now we attempt to generalize this idea in the quantum chromodynamic case. However, study of the problem of confinement is beyond the scope of the given work and we therefore include only some general remarks. Within the framework of our model, equations of motion for gluon and quark fields may be given, in momentum representation, as

$$p^2 \cosh^{-2}[l(-p^2)^{1/2}] G_\mu^a(p) = 0$$

$$(\hat{p} - m) \cosh^{-1}[l(m^2 - p^2)^{1/2}] q^a(p) = 0$$

Here we have omitted the color indices and assume that gluon is a massless particle and quark is a massive one with mass m. It is a common requirement that they do not exist in free states $p^2 = 0$ and $p^2 = m^2$. This statement is satisfied in our case due to the factor $\cosh^{-2} x$ (or $\cosh^{-1} x$) which ensures nontrivial solutions $G^a(p) \neq 0$ and $q^a(p) \neq 0$, where the upper index indicates that these fields are hidden inside a domain characterized by length a, for example, a bag with radius a. In our case this value may be identified with the parameter l of the theory.

Further, we propose that due to the presence of this domain (possibly quantized), translation invariance is not conserved in the theory of the quark and gluon fields and therefore their propagators acquire the form

$$D^a(x) = \frac{1}{(2\pi)^4} \int d^4p \frac{e^{ip(x-a)}}{-p^2 - i\varepsilon} \cosh^{-2}[l(-p^2)^{1/2}]$$

$$S^a(x) = \frac{1}{(2\pi)^4} \int d^4p \frac{e^{ip(x-a)}}{m^2 - p^2 - i\varepsilon} (\hat{p} + m) \cosh^{-1}[l(m^2 - p^2)^{1/2}]$$

where a_μ is a four-vector with components $a_\mu = (a_0 = 0, \mathbf{a})$, $|\mathbf{a}| = 1$. In the given case, the Coulomb-type potential between quarks takes the form

$$U_G(r) = \frac{g^2}{(2\pi)^3} \int d^3p \frac{e^{i\mathbf{p}(\mathbf{r}-\mathbf{a})}}{\mathbf{p}^2} \cosh^{-2}[l(\mathbf{p}^2)^{1/2}]$$

Here it is assumed that $\mathbf{a} = \mathbf{n}a$, where $\mathbf{n}$ is the unit vector. Taking into account our calculation carried out above, we have

$$U_G(r) = \begin{cases} -\dfrac{g^2}{2\pi^2 l} - \dfrac{g^2}{144 l^3} |\mathbf{r} - \mathbf{a}|^2 & \text{for } |\mathbf{r} - \mathbf{a}| < l \\ \dfrac{g^2}{4\pi |\mathbf{r} - \mathbf{a}|} & \text{for } l < |\mathbf{r} - \mathbf{a}| \end{cases}$$

The general behavior of this potential is sketched in Figure 5, from which we see that quarks are indeed hidden within the domain characterized by the length of confinement.

Now we determine the energy scales at which different forces are unified. It is also possible in our model. For this purpose, at the beginning we construct the weak potential between two particles. In accordance with the old traditional method we do this in the following manner:

$$D_w(p) = (g_{\mu\nu} - k_\mu k_\nu / m_w^2)(m_w^2 - p^2 - i\varepsilon)^{-1} \underset{m_w \to \infty}{\Longrightarrow} \frac{g_{\mu\nu}}{m_w^2}$$

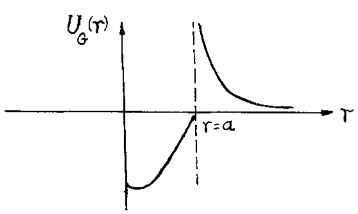

Fig. 5. General behavior of the potential between two quarks in the theory of quantized space-time.

or in the language of the Feynman diagrams .(see Figure 6). At this, the weak potential acquires the form

$$U_w^{\text{loc}}(r)=\frac{g_w^2}{(2\pi)^3}\int d^3p\frac{e^{i\mathbf{pr}}}{m_w^2+\mathbf{p}^2}\left(\delta_{ij}-\frac{P_iP_j}{m_w^2}\right)\Rightarrow\frac{G_F}{\sqrt{2}}\frac{\delta_{ij}}{(2\pi)^3}\int d^3p\, e^{i\mathbf{pr}}$$

where $G_F/\sqrt{2}=g_w^2/m_w^2$. It is a local case. In our model it takes the form

$$\begin{aligned}U_w(r)&=\frac{G_F}{\sqrt{2}}\frac{1}{(2\pi)^3}\int d^3p\, e^{i\mathbf{pr}}\cosh^{-2}[l(\mathbf{p}^2)^{1/2}]\\&=\frac{G_F}{\sqrt{2}}\frac{1}{2\pi^2rl^2}\int_0^\infty dy\, y\sin\left(\frac{r}{l}y\right)\cosh^{-2}y\\&=-\left(\frac{G_F}{\sqrt{2}\,2\pi^2rl^2}\right)\frac{d}{dz}\left[\frac{\pi z}{2\sinh(\frac{1}{2}\pi z)}\right]\\&=-\frac{G_F}{\sqrt{2}}\frac{1}{4\pi rl^2}\left[\frac{1}{\sinh(\frac{1}{2}\pi z)}-\tfrac{1}{2}\pi z\frac{\cosh(\frac{1}{2}\pi z)}{\sinh^2(\frac{1}{2}\pi z)}\right]\end{aligned}$$

where $z=r/l$. It is easy to verify that this potential is finite at the point $r=0$. For $r\to 0$, we have

$$U_w(r)=\frac{G_F}{\sqrt{2}}\frac{1}{24l^3}-\frac{G_F}{\sqrt{2}}\frac{\pi^2}{2^5l^5}\frac{31}{180}r^2 \tag{74}$$

We now assume that the electrostatic energy $U_C(0)$ and the weak-static energy $U_w(0)$ of the electron coincides with the absolute value at the energy scale given by $E_{\text{ew}}=\hbar/l_{\text{ew}}c$. Here we call E_{ew} the electroweak energy scale at which electromagnetic and weak interactions are unified. Thus, from (32) and (74), we have

$$\frac{e^2}{2\pi^2l_{\text{ew}}}=\frac{G_F}{\sqrt{2}}\frac{1}{24l_{\text{ew}}^3}\qquad\text{or}\qquad E_{\text{ew}}=(\alpha/G_F)^{1/2}(48\sqrt{2}/\pi)^{1/2}=118.1\text{ GeV}$$

where $\alpha=e^2/4\pi$ and $E_{\text{ew}}=\hbar/l_{\text{ew}}c$. We see that the obtained energy scale is closer to the unified scale of electroweak interactions due to S. Weinberg, A. Salam, and Sh. Glashow, i.e., it coincides with the mass of $W^\pm$ and Z^0 bosons.

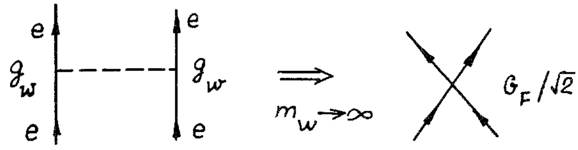

Fig. 6. The illustration of the passage of the intermediate vector weak interaction into the four-fermion weak interaction.

Analogously, comparing the weak-static energy $U_w(0)$ and strong Yukawa energy values $U_Y(0)$ at the same energy scale $E_{\rm nw} = \hbar / l_{\rm nw} c$ (we call it the nuclear-weak energy scale), we have from (49) and (74):

$$\frac{g^2}{2\pi^2 l_{\rm nw}} = \frac{G_F}{\sqrt{2}} \frac{1}{24 l_{\rm nw}^3} \qquad \text{or} \qquad E_{\rm nw} = (f/G_F)^{1/2}(48\sqrt{2}/\pi)^{1/2} = 5353 \text{ GeV}$$

where $f = g^2/4\pi \sim 15$, $E_{\rm nw} = \hbar / l_{\rm nw} c$.

It is interesting to notice that the hypothesis of quantized space-time may indicate the energy scale of a grand unified theory linking weak, strong, and electromagnetic interactions at very high energy. It is no exception that this unification takes place at the scale of energy $E_{\rm nw} = 5353$ GeV (or, equivalently at the distances $l = 4 \times 10^{-18}$ cm) which is must lower than the energy scale 10^{15} GeV discussed in the grand unified theory.

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